Year 12 Mathematics Term II Examination 2007

QUESTION 1. **Marks**

(a) Solve for x : $-2x^2 + 1 \geq -31$ **2**

(b) If α and β are the roots of the equation $4x^2 - 3x + 5 = 0$ find :

(i) $\alpha + \beta$ **1**

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **2**

(iii) $\alpha^2 + \beta^2$ **2**

(c) (i) State the condition for equal roots of a quadratic equation. **1**

(ii) The line with equation $y = mx - 9$ is a tangent to the parabola with equation $y = x^2$. Find the values of *m*. **2**

(d) The acceleration $\ddot{x}$ of a particle is given by $\ddot{x} = 15\sqrt{t}$, where the displacement is *x* metres and time *t* seconds. If the particle is initially 4 metres to the left of the origin and has an initial velocity of -3 metres per second, find the velocity and displacement functions in terms of time *t*. **2**

(e) Two dice each with the numbers 2, 4, 6, 8, 10 and 12 on their faces are thrown. By using a dot diagram or otherwise, find the probability that the sum is greater than 10. **3**

QUESTION 2.

(a) Write a quadratic equation with the sum of roots equal to 6 and the product of roots equal to -4. **2**

A particle moves horizontally in a straight line. The displacement *x* metres as a function of time *t* seconds is given by $x = 4t^3 - 15t^2 - 18t - 6$

(b) (i) Find the initial displacement, initial velocity and initial acceleration. **3**

(ii) Find the time and displacement when the particle comes to rest. **3**

(iii) Determine the direction of motion after the particle comes to rest. Justify your answer. **2**

As a sliding door closes in time *t* seconds, the door opening *x*cm is given by $x = Ae^{-kt}$.

(c) (i) Show that the rate of closure of the door $\dfrac{dx}{dt}$ is proportional to the size of the door opening *x*. **1**

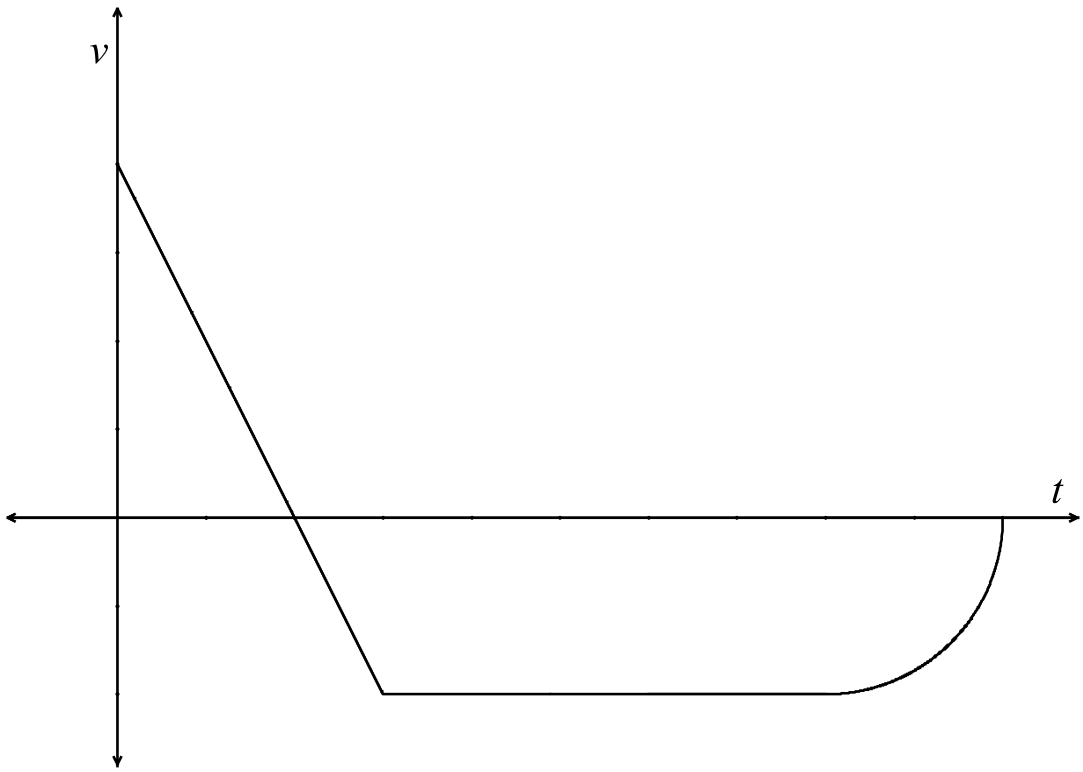
(ii) If the initial opening is 80 cm and the initial speed of closure is 10 cm/s, find the values of *A* and *k*. **2**

(iii) Find the time(correct to 2 decimal places) for the door to be 80% closed. **2**

QUESTION 3. **Marks**

(a) The velocity v m/s against time t seconds graph shown is comprised of straight lines and a circular arc.

(i) Copy the graph, hence graph directly beneath using the same time scale **:** **1**



(ii) (α) the acceleration function $\ddot{x}$ against time t for $0 \le t \le 10$. **2**

(β) the displacement function x against time t for $0 \le t \le 10$ if the particle is initially at the origin .(Use scale 1cm= 2m) **3**

(b) A car travels at 100 km/hr along a straight horizontal road at night. Posts are placed at 5m intervals along the left side of the road.

(i) Find the speed of the car in metres per second (correct to 2 decimal places). **1**

(ii) A truck with bright lights approaches the car from the opposite direction and blinds the driver of the car for 2 seconds.

(α) Find the distance the car travels while the driver is blinded, hence show that the number posts N that the car could pass is 12. **2**

If the probability of not hitting a post is $\frac{99}{100}$, find the probability of **:**

(β) not hitting the first three posts (4 decimal places). **2**

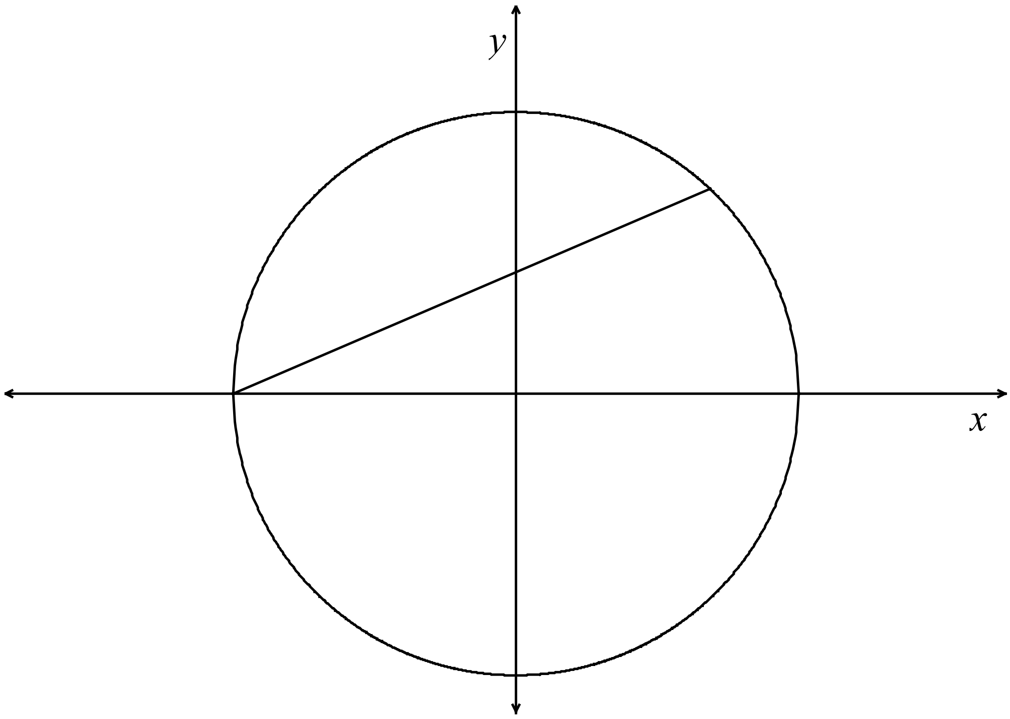
(γ) hitting at least one post while the driver is blinded (4 decimal places). **2**

QUESTION 4.

(a) The displacement function of a particle is given by **:** $x = \sqrt{1+t^2}$. **5**

By finding $\ddot{x}$ show that the direction of the force applied to the particle never changes.

(b)



The chord shown cuts the circle with equation $x^2 + y^2 = 1$ at the points *A* and *P*.

(i) Find the equation in gradient intercept form, of the chord *AP* with gradient *m*. **1**

(ii) Show that the *x* values of the intersection points *A* and *P* satisfy the equation **2**
$\left(m^2+1\right)x^2 + 2m^2x + m^2 - 1 = 0$

(iii) Find the co-ordinates of the intersection point *P*. **3**

(iv) If $\angle PAO = \alpha$ show $\angle POB = 2\alpha$, giving reasons. **2**

(v) Hence find an expression for $\tan 2\alpha$ in terms of $\tan\alpha$. **2**

End Of Examination

YR12 – 2007 – Term 2 – 2U MATHEMATICS

SOLUTIONS

(a) $-2x^2+1 \geq -31$

$-2x^2 \geq -32$

$x^2 \leq 16$

$-4 \leq x \leq 4$ (1)

b) (i) $\alpha+\beta = \frac{3}{4}$ (1)

(ii) $\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta}$

$= \frac{\frac{3}{4}}{\frac{5}{4}}$ (1)

$= \frac{3}{5}$ (1)

(iii) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ (1)

$= \left(\frac{3}{4}\right)^2 - 2\times\frac{5}{4}$

$= -\frac{31}{16}$

$= -1\frac{15}{16}$ (1)

c) (i) $\Delta = 0$ 1

(ii) $x^2 = mx - 9$

$x^2 - mx + 9 = 0$

$\Delta = 0$

$m^2 - 36 = 0$ (1)

$m = \pm 6$ (1)

d) $\ddot{x} = 15\sqrt{t}$

$\dot{x} = 10t^{3/2} + c$

$\dot{x} = -3 \quad t = 0$

$-3 = 0 + c$

$c = -3$

$\dot{x} = 10t^{3/2} - 3$ (1)

$x = 4t^{5/2} - 3t + c$

$x = -4 \quad t = 0 \Rightarrow c = -4$

$\therefore x = 4t^{5/2} - 3t - 4$ (1)

(e)

	2	4	6	8	10	12
2					'	'
4				'	'	'
6			'	'	'	'
8		'	'	'	'	'
10	'	'	'	'	'	'
12	'	'	'	'	'	'

(1)

$P(\text{sum} > 10) = \frac{26}{36} = \frac{13}{18}$ (1) (1)

2(a) $x^2 - 6x - 4 = 0$ (1)

(b) $x = 4t^3 - 15t^2 - 18t - 6$

$v = 12t^2 - 30t - 18$

$\ddot{x} = 24t - 30$

(i) $t = 0$ $x = -6$ m [6 m left 0] (1)

$v = -18$ m/s [18 m/s left] (1)

$\ddot{x} = -30$ m/s^2 (1)

(ii) $v = 0$

$12t^2 - 30t - 18 = 0$

$2t^2 - 5t - 3 = 0$

$(2t+1)(t-3) = 0$ (1)

$t = -\frac{1}{2}$ or $t = 3$

But $t > 0$ $\therefore t = 3$ s only (1)

$x = -87$ m (1)

(iii) $\ddot{x} = 24\times 3 - 30$

$= 42$

> 0 (1)

$\therefore$ Force $m\ddot{x} > 0$ as $m > 0$, $\ddot{x} > 0$

Particle moves right from rest, never stops or changes direction after $t > 3$ (1)

2c (i) $x = Ae^{-kt}$

$\frac{dx}{dt} = -kAe^{-kt}$

$= -kx$ *

$\therefore \frac{dx}{dt} \propto (-x)$.

$t = 0 \quad x = 80 \Rightarrow 80 = Ae^{0}$

$A = 80$ ①

Also from * $\quad -10 = -k \cdot 80$

$k = \frac{1}{8}$

$\therefore x = 80e^{-\frac{t}{8}}$

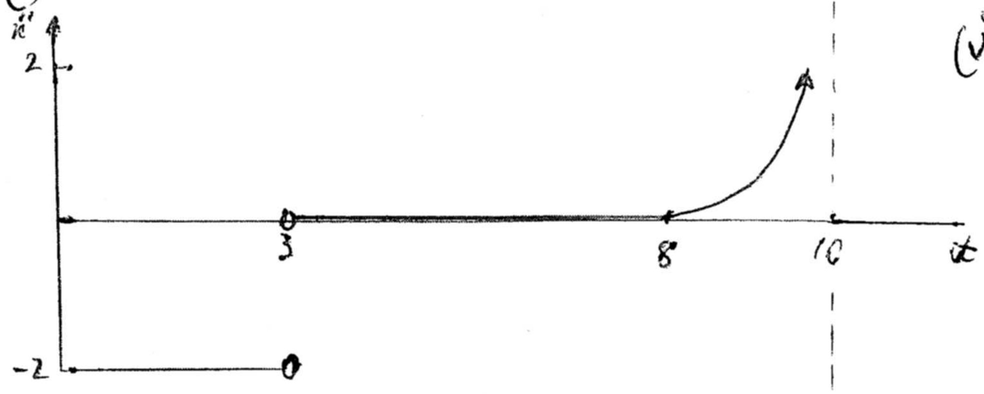
(ii) $0{\cdot}2A = Ae^{-\frac{t}{8}}$

$\frac{t}{8} = -\ln 0{\cdot}2$

$t \doteqdot 12{\cdot}88\,s$

3(a) (i) ①

(ii)



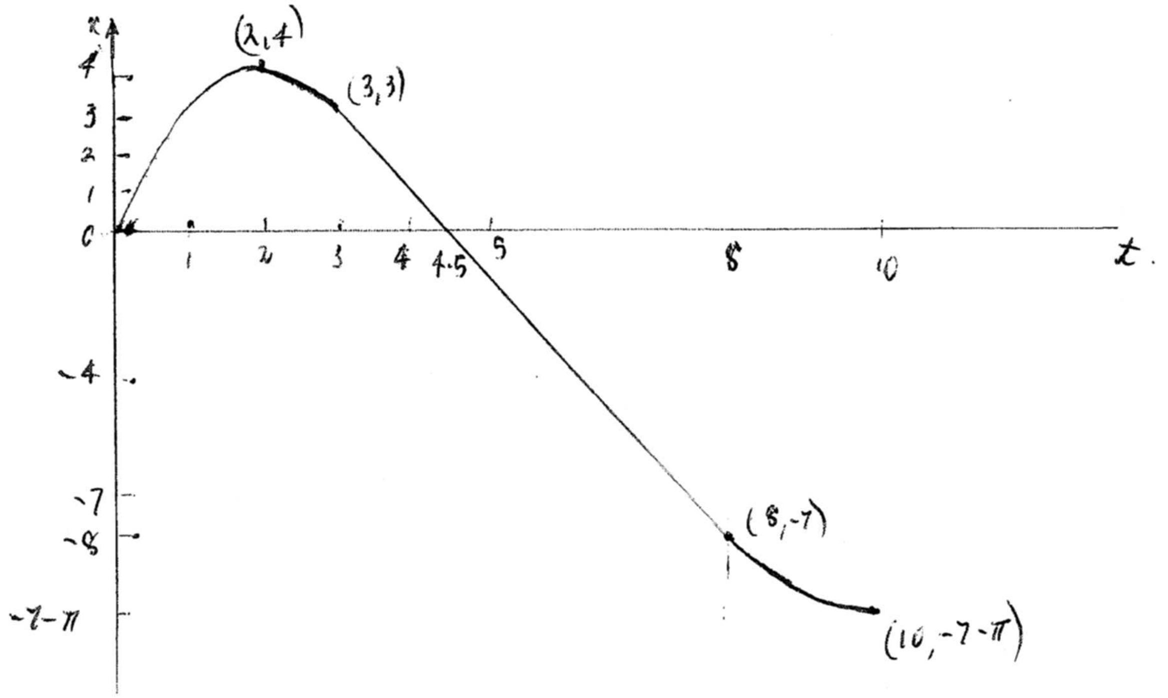
(iv) Time return to origin $4{\cdot}5\,s$

(v) Average speed

$= \frac{2+4+7+\pi}{10}$

$\doteqdot 1{\cdot}8\,m/s$

(iii)



(b) (i) $v = 100 \times \frac{1000}{3600}$

$= 27.78$ m/s ①

(ii) (α) Distance travelled $= 2 \times 27.78$

$= 55.56$ m ①

Number posts $= \frac{55.56}{5} + 1$ ①

$= 12$ ②

(β) $P(\tilde{H}\tilde{H}\tilde{H}) = 0.99^3$ ①

$= 0.9703$ ①

(β δ) P(Hitting) $= 1 - $ P(Not Hitting) ①

$= 1 - 0.99^{12}$

$= 0.1136$ ①

4(a) $x = \sqrt{1+t^2}$

$\dot{x} = \frac{t}{\sqrt{1+t^2}}$ ②

$\ddot{x} = \frac{\sqrt{1+t^2}\cdot 1 - t \cdot \frac{t}{\sqrt{1+t^2}}}{1+t^2}$

$= \frac{1+t^2 - t^2}{(1+t^2)^{3/2}}$

$= \frac{1}{(1+t^2)^{3/2}}$

> 0 for all $t > 0$ ②

Since $F = m\ddot{x}$ $m > 0$

$\therefore$ Direction force always > 0

and never equals zero ①

$\therefore$ Force never changes direction

(b) (i) $\angle PAO = \alpha$ given

$\angle OPA = \alpha$ [equal angles opposite equal sides/radii in ΔAPO] (1)

$\angle POB = \angle PAO + \angle OPA$ [Exterior angle triangle is equal to the sum of the two interior opposite angles] (1)

$= \alpha + \alpha$

$= 2\alpha$

(ii)

$$y - y_1 = m(x - x_1)$$

$$y - 0 = m(x + 1)$$

$$y = mx + m \quad (1)$$

(iii)

$$x^2 + (mx + m)^2 = 1 \quad (1)$$

$$x^2 + m^2x^2 + 2m^2x + m^2 = 1 \quad (1)$$

$$(1 + m^2)x^2 + 2m^2x + m^2 - 1 = 0$$

$$(x + 1)\left((1 + m^2)x + m^2 - 1\right) = 0 \quad (1)$$

$$x = -1 \quad \text{or} \quad x = \frac{1 - m^2}{1 + m^2} \quad (1)$$

But $x \neq -1$ $\therefore$ $x_P = \dfrac{1 - m^2}{1 + m^2}$

$$y_P = m\left(\frac{1 - m^2}{1 + m^2}\right) + m$$

$$= \frac{m - m^3 + m^3 + m}{1 + m^2}$$

$$= \frac{2m}{1 + m^2} \quad (1)$$

$$P \equiv \left(\frac{1 - m^2}{1 + m^2}, \frac{2m}{1 + m^2}\right)$$

(v) $\tan 2\alpha = \dfrac{y_P}{x_P}$

$$= \frac{2m}{1 - m^2} \quad (1)$$

But $m = \tan\alpha$

$$\therefore \tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} \quad (1)$$